Supplementary Materials

I. Computational Complexity Analysis

First, note that the computational complexity of the matrix multiplication for two matrices $\boldsymbol{M}_{n\times m}$ and $\boldsymbol{M}_{m\times p}$ is $O(nmp)$ using the definition of matrix multiplication, and of the matrix inverse for a matrix $\boldsymbol{M}_{m\times m}$ is $O(m^3)$ using the Gauss–Jordan elimination method. (The results can be improved by advanced algorithms, e.g., the Strassen algorithm.) Therefore,

1) in the time-update step, the computational complexity of (18) is $O[(n+m)\times n+(n+m)]$, of (19) is $O[(n+m)\times n\times n+(n+m)\times n\times(n+m)+(n+m)\times(p+m)\times(n+m)+2\times(n+m)\times(n+m)]$;
2) in the step of obtaining the worst-case scenario, the computational complexity of (33) is $O[n+m]$, of (47) is $O[2\times(n+m)\times(n+m)]$;
3) in the measurement-update step, the computational complexity of (35) is $O[n+m^3+nmm+nm+n]$, of (36) is $O[n^2+m^3+nmm+nmn+n^2]$.

Let $d := \max\{n,m,p\}$. As a result, the computational complexity of Algorithm 1 is asymptotically $O(d^3)$. Since for a usual state estimation problem, $n \geq p$ and $n \geq m$, the computational complexity of Algorithm 1 is $O(n^3)$.

II. Supplementary Experiments

In Introduction I, we claim that the proposed method would be robust against uncertainties due to violation(s) of the following three types of assumptions.

4) $\boldsymbol{\mu}^{\boldsymbol{w}}_k$, $\boldsymbol{\mu}^{\boldsymbol{v}}_k$ are exactly known and typically $\boldsymbol{\mu}^{\boldsymbol{w}}_k \equiv \boldsymbol{0}$, $\boldsymbol{\mu}^{\boldsymbol{v}}_k \equiv \boldsymbol{0}$;
5) $\boldsymbol{Q}_k$ and $\boldsymbol{R}_k$ are exactly known;
6) $\boldsymbol{F}_{k-1}$, $\boldsymbol{G}_{k-1}$, and $\boldsymbol{H}_k$ are exactly known.

In the experiment section (Section VI), we have studied the robustness of the proposed method against uncertainties in the system matrix $\boldsymbol{F}_k$ [i.e., the type 6)]. In this appendix, we investigate the robustness of the proposed method against uncertainties in the statistical properties, i.e., mean and covariance, of the noises, respectively. Comparisons are made with the non-robust canonical Kalman filter.

First, we suppose the mean vector $\boldsymbol{\mu}^{\boldsymbol{w}}_{k-1}$ of the process noise $\boldsymbol{w}_{k-1}$ is not exactly zero-valued [i.e., the type 4)]. In this case, the underlying true system is

$$\begin{cases} \boldsymbol{x}_k &= \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}_{k-1}\boldsymbol{d}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{w}_{k-1}, \\ \boldsymbol{y}_k &= \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k, \end{cases}$$

while the nominal system is

$$\begin{cases} \boldsymbol{x}_k &= \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{w}_{k-1}, \\ \boldsymbol{y}_k &= \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k, \end{cases}$$

where we use $\boldsymbol{\Gamma}_{k-1}\boldsymbol{d}_{k-1} := \boldsymbol{G}_{k-1}\boldsymbol{\mu}^{\boldsymbol{w}}_{k-1}$ to model the uncertain mean of $\boldsymbol{w}_{k-1}$ (strictly, the uncertain mean of $\boldsymbol{G}_{k-1}\boldsymbol{w}_{k-1}$). Without loss of generality, we still use the nominal values of $\boldsymbol{F}_{k-1}$, $\boldsymbol{G}_{k-1}$, $\boldsymbol{H}_k$, $\boldsymbol{Q}_{k-1}$, and $\boldsymbol{R}_k$ in Section VI. Besides, we assume $\boldsymbol{\Gamma}_{k-1} := [1,0]^T$ and $\boldsymbol{d}_{k-1}$ is a random variable which follows a standard Gaussian distribution. We have results in Fig. 1 (see also its caption for RMSEs), through which the robustness of the proposed method against uncertainties in the mean of the noises is validated.

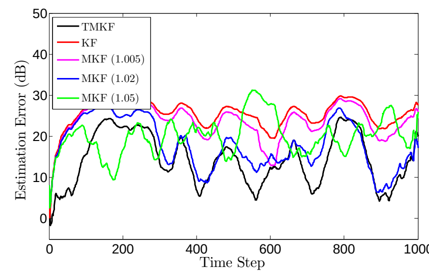

Fig. 1. Results with $\gamma_2 = 1.005$, $\gamma_2 = 1.02$, and $\gamma_2 = 1.05$, respectively. RMSE: TMKF = 9.43, KF = 21.96, MKF (1.005) = 19.34, MKF (1.02) = 13.25, MKF (1.05) = 18.28.

Second, we suppose the covariance matrices $\boldsymbol{Q}_{k-1}$ of the process noise $\boldsymbol{w}_{k-1}$ and $\boldsymbol{R}_k$ of the measurement noise $\boldsymbol{v}_k$ are not exactly known [i.e., the type 5)]. In this case, the underlying true system and the nominal system are

$$\begin{cases} \boldsymbol{x}_k &= \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{G}_{k-1}\boldsymbol{w}_{k-1}, \\ \boldsymbol{y}_k &= \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k, \end{cases}$$

but they have different $\boldsymbol{Q}_{k-1}$ and $\boldsymbol{R}_k$. We investigate a target tracking problem discussed in [1]. Therefore, the nominal values of $\boldsymbol{F}_{k-1}$, $\boldsymbol{G}_{k-1}$, $\boldsymbol{H}_k$, $\boldsymbol{Q}_{k-1}$, and $\boldsymbol{R}_k$ are:

$$\boldsymbol{F}_{k-1} := \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}, \qquad \boldsymbol{G}_{k-1} := \begin{bmatrix} (\Delta t)^2/2 \\ \Delta t \end{bmatrix},$$

$$\boldsymbol{H}_k := \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad \boldsymbol{Q}_{k-1} := 0.1, \quad \boldsymbol{R}_k := 20^2,$$

where $\Delta t := 1$ (second) is the sampling time. The true values of $\boldsymbol{Q}_{k-1}$ and $\boldsymbol{R}_k$ are

$$\boldsymbol{Q}_{k-1} := 0.1 + 0.1 \times q_{k-1}, \quad \boldsymbol{R}_k := 20^2 + 10 \times r_k,$$

where q_{k-1} and r_k are two random variables following standard uniform distributions in $[0,1]$. We have results in Fig. 2 (see also its caption for RMSEs), through which the robustness of the proposed method against uncertainties in the covariances of the noises is validated.

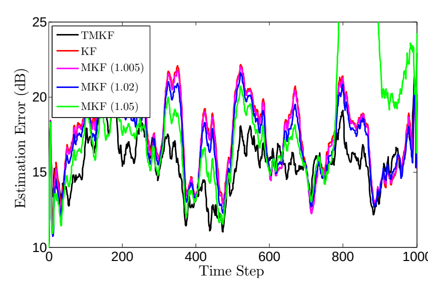

Fig. 2. Results with $\gamma_2 = 1.005$, $\gamma_2 = 1.02$, and $\gamma_2 = 1.05$, respectively. RMSE: TMKF = 10.16, KF = 12.65, MKF (1.005) = 12.43, MKF (1.02) = 11.88, MKF (1.05) = 489.86.

References

[1] Y. Huang, Y. Zhang, Z. Wu, N. Li, and J. Chambers, “A novel adaptive kalman filter with inaccurate process and measurement noise covariance matrices,” *IEEE Transactions on Automatic Control*, vol. 63, no. 2, pp. 594–601, 2017.